

GUIDED NOTES: Creating Probability Simulations

A **simulation** consists of a collection of things that **happen at random**. There is a situation that is **repeated** a large number of times, called the **component** of the simulation. Each component has a set of possible outcomes.

Example: When flipping a coin, it is random whether it will land on the heads or a tails side of the coin. Use a simulation and the chart below to determine the components and whether or not you would expect to the coin to land with the heads side of the coin up three times in a row or not.

| Trial # | Result |
|---------|----------------------------------------|
| 1 | H T H |
| 2 | H T T |
| 3 | H H H ★ |
| 4 | T H H |
| 5 | T T H |
| 6 | H H T |
| 7 | H T T |
| 8 | H T H |

Component - flipping 3 coins & recording outcome

In your trial, what was the probability that you flipped a coin three times and it landed with heads up each time?

$$\frac{1}{8} \rightarrow 12.5\%$$

If you were to flip 60 trials of coins, would you expect this probability to change? Let's try!

Yes, it would get closer to the actual.

Example: Fifty-seven students participate in a lottery for a particularly desirable dorm. Twenty of the participants were members of the same varsity team. All three winners were members of the team. Use a simulation to determine whether an all-team outcome could reasonably be expected to happen.

The component here is the selection of a student for the room. Since there are 57 students in the drawing, let's use 1 - 57 to represent the students. Let's use 1 - 20 represent the team members and 21 - 57 represent the rest of the students.

#1-20: football players

#21-57: students

Run RandInt (1, 57, 3) to simulate drawing names. (eliminate any trials where the same number comes up more than once). If all three numbers are between 1 and 20, then the whole room goes to team members.

RandInt(lower bound, upper bound, #)

You may get something like this:

| | | |
|------------------------------------------------------------------------------------------------------------|----|-----------------------------------------------------------------------------------------|
| RandInt (1, 57, 3) 23, 51, 19 1 team members, 2 non-members this counts as "not all team members" | OR | RandInt (1, 57, 3) 5, 19, 7 all team members this counts as "all team members" |
|------------------------------------------------------------------------------------------------------------|----|-----------------------------------------------------------------------------------------|

When you run the trial once, it gives you one possible result, but that's not enough to make a decision. It will take lots of trials to decide whether an all-team outcome would be reasonable. Let's run 10 trials and look at the results:

| Trial # | Numbers | Result |
|---------|------------|----------------------|
| 1 | 14, 28, 56 | only one team member |
| 2 | 4, 47, 23 | 2 |
| 3 | 19, 15, 1 | 3 |
| 4 | 45, 32, 11 | 1 |
| 5 | 6, 18, 35 | 2 |
| 6 | 11, 51, 23 | 2 |
| 7 | 42, 27, 20 | 1 |
| 8 | 22, 45, 51 | 0 |
| 9 | 55, 38, 29 | 0 |
| 10 | 6, 22, 54 | 1 |

$\frac{1}{10} \rightarrow 10\%$

Looking at these results, there is 1 trial out of 10 that has the room going to three team members, so the probability would be 10%. Ten trials really isn't enough to make a decision either. It usually takes several hundred trials to get an accurate picture of the situation.

After 100 trials, results could look like this:

| Room Selection | frequency |
|----------------------|-----------|
| all team members | 6 |
| not all team members | 94 |

$\frac{6}{100} \rightarrow 6\%$

Since the simulation shows that there is a 6% chance that the room will be filled by all team members, it is pretty surprising that this occurred. You take a quiz with 6 multiple choice questions. Each question has 4 possible answers. Unfortunately, you forgot there was a quiz today, so you didn't study at all, so you have to guess at the answers. Design a simulation for this situation and determine the probability of getting at least half of the questions right.

First, figure out the probabilities we're working with.

$$P(\text{guessing right}) = \frac{1}{4} \quad P(\text{guessing wrong}) = \frac{3}{4}$$

Now we have to assign numbers to use in our simulation that will have the same ratio as these probabilities. Since there are 4 options, use the digits 1 – 4. Let one number represent the correct answer, and the other three will represent the wrong answers.

$$1 = \text{right answer} \quad 2, 3, 4 = \text{wrong answers}$$

Now we will run a random integer generator to simulate one try at the quiz. Since there are 6 questions on the quiz, we need 6 numbers. Run $\text{RandInt}(1, 4, 6)$ – this will give us 6 numbers between 1 and 4.

| Trial # | Numbers | Number of Right Answers |
|---------|---------------------|-------------------------|
| 1 | 1, 3, 2, 3, 4, 2 | 1 |
| 2 | 3, 4, 4, 2, 1, 1, 4 | 2 |
| 3 | 4, 2, 3, 1, 3, 3 | 1 |
| 4 | 3, 3, 1, 1, 1, 4 | 3 |
| 5 | 1, 3, 4, 3, 1, 2 | 2 |
| 6 | 3, 1, 3, 1, 3, 2 | 2 |
| 7 | 1, 2, 4, 1, 3, 4 | 2 |
| 8 | 2, 4, 4, 2, 4, 3 | 0 |
| 9 | 4, 3, 3, 3, 2, 4 | 0 |
| 10 | 2, 1, 4, 3, 2, 2 | 1 |

What percentage of the trials had at least three answers correct?

10%