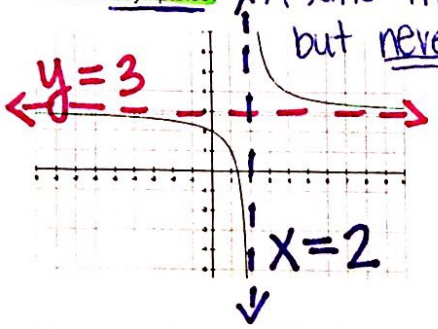
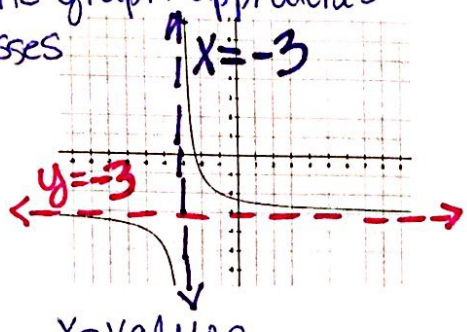
GUIDED NOTES: Asymptotes of Rational Functions

What are asymptotes? A line that the graph approaches but never crosses





DOMAIN: The domain of a function is the set of all possible x-values

Restrictions are values not included in the domain!

Vertical Asymptotes:

To find vertical asymptotes: ① Factor/set denominator = 0 ② use what doesn't cancel

EX1. $y = \frac{x-3}{(x+2)(x-2)}$

$x+2=0$ $x-2=0$

$x=-2$ $x=2$

V.A @ $x=-2, 2$

EX2. $y = \frac{x-1}{x^2+5x+4} = \frac{x-1}{(x+1)(x+4)}$

$x+1=0$ $x+4=0$

$x=-1$ $x=-4$

V.A @ $x=-1, -4$

Holes:

What is the Vertical Asymptote of $y = \frac{x(x-4)}{x-4}$? Graph it, what do you notice? No V.A

A HOLE in the graph is when (x – a) is a factor in both the numerator and the denominator (it cancels out!!)

EX3. $y = \frac{x^2-1}{x^2-2x-3} = \frac{(x+1)(x-1)}{(x+1)(x-3)}$

$x+1=0$ $x+3=0$

$x=-1$ $x=-3$

Hole @ $x=-1$

V.A @ $x=-3$

EX4. $y = \frac{12x+24}{x^2+2x} = \frac{12(x+2)}{x(x+2)}$

what's left over: $x=0$ $x+2=0$, $x=-2$

what cancels

Hole @ $x=-2$

V.A @ $x=0$

Horizontal Asymptotes:

To find a horizontal asymptotes, we focus on the Degree (highest exponent) of the numerator and the denominator!

① $N = D$: H.A @ y = coefficients

② $N < D$: BOBO! Bigger on Bottom = 0 H.A @ $y=0$

③ $N > D$: No H.A

Examples:

EX5. $y = \frac{6x-5}{3x+9}$ Degree 1 / Degree 1

H.A @ $y = \frac{6}{3} = 2$

EX6. $y = \frac{4x^6+3x^4-4x^2}{3x}$ Degree 6 / Degree 1

No H.A

EX7. $y = \frac{(x-3)^1(x+4)^2}{(x-5)^3(x+2)^2}$ $1+2=3$, $3+2=5$ Degree 3 / Degree 5

H.A @ $y=0$

EVERYTHING ALL TOGETHER:

Find the holes, vertical asymptotes, domain, and horizontal asymptotes for each rational function.

EX8. $y = \frac{5x^2+20x}{x^2+11x+28} = \frac{5x(x+4)}{(x+4)(x+7)}$

$x+4=0$ $x+7=0$

$x=-4$ $x=-7$

Hole @ $x=-4$

V.A @ $x=-7$

Domain: All real #'s except $x \neq -4, -7$

H.A @ $y=5$

Degree 2 / Degree 2 . $\frac{5}{1}$

EX9. $y = \frac{2x^2+10x+12}{x^3+3x^2+2x} = \frac{2(x^2+5x+6)}{x(x^2+3x+2)} = \frac{2(x+2)(x+3)}{x(x+1)(x+2)}$

$x=0$ $x+1=0$ $x+2=0$

$x=-1$ $x=-2$

Hole @ $x=-2$

V.A @ $x=0, -1$

Domain: All real #'s except $x \neq -1, -2, 0$

H.A @ $y=0$

Degree 2 / Degree 3