

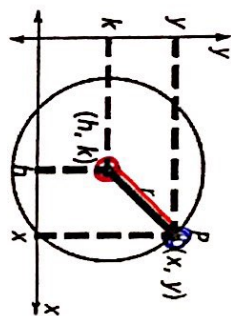
6.10 Equations of Circles

SWBAT graph circles on the coordinate plane and write the equations of circles in standard form.

Standard Form of Circles

$$(x-h)^2 + (y-k)^2 = r^2$$

Center: (h, k) **Radius:** r **Point on the circle:** (x, y)



Example 1: Write the equation of a circle with the given information.

a) Center (0,0), Radius=10

$h = 0$ $k = 0$ $r = 10$

$$(x-0)^2 + (y-0)^2 = 10^2$$

$$x^2 + y^2 = 100$$

Example 2: Determine the center and radius of a circle the given equation.

a) $x^2 + y^2 = 9$ $\sqrt{9} = \frac{\sqrt{9}}{\sqrt{1}} = 3$ b) $(x+3)^2 + (y-5)^2 = 81$

Center: (0,0) r: 3 Center: (-3,5) r: 9

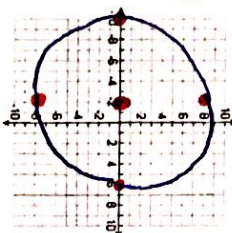
c) $(x+4)^2 + (y+6)^2 = 1$ Center: (-4,-6) r: 1

Example 3: Use the center and the radius to graph each circle.

a) $(x+2)^2 + y^2 = 64$

Center: (-2,0)

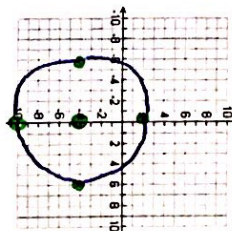
Radius: 8



b) $x^2 + (y+4)^2 = 36$

Center: (0,-4)

Radius: 6



Example 4: Write the equation of a circle with a given center (2, 5) that passes through the point (5, -1).

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(5-2)^2 + (-1-5)^2 = r^2$$

$$(3)^2 + (-6)^2 = r^2$$

$$9 + 36 = r^2$$

$$45 = r^2$$

$$(x-2)^2 + (y-5)^2 = 45$$

Center: (2, 5)

r: $\sqrt{45} = 3\sqrt{5}$

Writing an Equation with Two Points on the Circle

Find the midpoint (radius) between the two endpoints, and then follow steps 1-4.

Midpoint Formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Example 5: Write the equation of a circle with endpoints of diameter at (-6, 5) and (4, -3).

Midpoint: $\left(\frac{-6+4}{2}, \frac{5-3}{2} \right) = (-1, 1) \leftarrow$ Center

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(4+1)^2 + (-3-1)^2 = r^2$$

$$(5)^2 + (-4)^2 = r^2$$

$$41 = r^2$$

$$(x+1)^2 + (y-1)^2 = 41$$

Writing the Equation of a Circle in Standard Form

Step 1:	Group x's and group y's together.
Step 2:	Move any constants to the right side of the equation.
Step 3:	Use complete the square to make a perfect square trinomial for the x's and then again for the y's. *Remember, whatever you do to one side of the equation, you must do to the other!
Step 4:	Simplify factors into standard form of a circle!

Example 5: Write the equation of a circle in standard form. Then, state the center and the radius.

a) $x^2 + y^2 + 4x - 8y + 16 = 0$

b) $x^2 + y^2 + 6x - 4y = 0$

c) $x^2 + y^2 - 6x - 2y + 4 = 0$

d) $x^2 + y^2 + 8x - 10y - 4 = 0$

Example 5 a | $x^2 + y^2 + 4x - 8y + 16 = 0$

$$(1) \quad \underbrace{x^2 + 4x}_{-16} + \underbrace{y^2 - 8y}_{-16} + 16 = 0$$

$$(2) \quad x^2 + 4x + y^2 - 8y = -16$$

$$(3) \quad x^2 + 4x + \frac{4}{\frac{4}{2} = (2)^2} + y^2 - 8y + \frac{16}{\frac{-8}{2} = (-4)^2} = -16 + 4 + 16$$

$$(4) \quad (x + 2)^2 + (y - 4)^2 = 4$$

* center: $(-2, 4)$ r: 2 *

b) $x^2 + y^2 + 6x - 4y = 0$

$$\underbrace{x^2 + 6x}_{9} + \underbrace{y^2 - 4y}_{4} = 0$$

$$x^2 + 6x + \frac{9}{\frac{6}{2} = (3)^2} + y^2 - 4y + \frac{4}{\frac{-4}{2} = (-2)^2} = 0 + 9 + 4$$

$$(x + 3)^2 + (y - 2)^2 = 13$$

center: $(-3, 2)$ r: $\sqrt{13}$