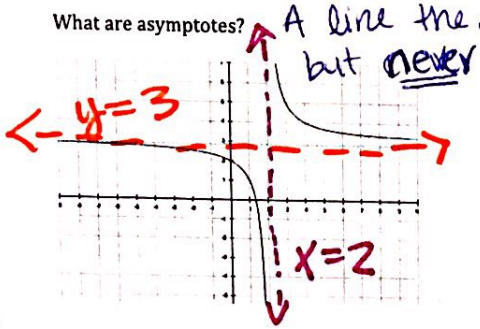
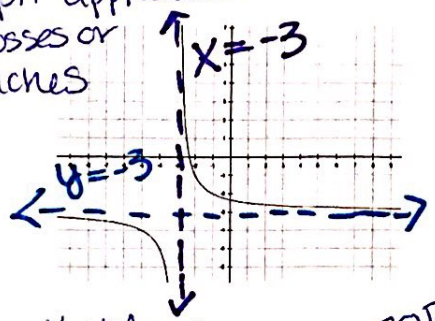
Asymptotes of Rational Functions

What are asymptotes? A line the graph approaches but never crosses or touches





DOMAIN: The domain of a function is the set of all possible X-values

Restrictions! are values not included in the domain!

RANGE: The range of a function is the set of all possible y-values

Restrictions! are values not included in the range!

BOBO

Bigger on Bottom Out!

Vertical Asymptotes:

To find Vertical Asymptotes ① set denominator = 0 ② ~~Restrictions that aren't~~ canceled

Examples:

1. $y = \frac{x-3}{(x+2)(x-2)}$ $\{x \neq 2, -2\}$ V.A@ $x=2$, $x=-2$

2. $y = \frac{x-1}{x^2+5x+4} = \frac{x-1}{(x+1)(x+4)}$ $\{x \neq -1, -4\}$ V.A@ $x=-1$, $x=-4$

Holes:

What is the Vertical Asymptote of $y = \frac{x(x-4)}{x-4}$? Graph it, what do you notice? No V.A, it cancels

A HOLE in the graph is when (x - a) is a factor **in both the numerator and the denominator** (It cancels out!!!)

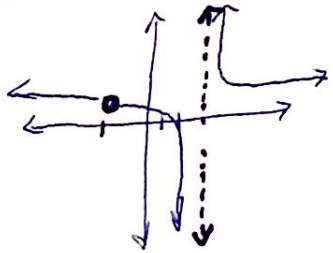
Examples:

1. $y = \frac{x^2-1}{x^2-2x-3} = \frac{(x+1)(x-1)}{(x+1)(x-3)}$

Restrictions $\{x \neq -1, 3\}$

Hole @ $x=-1$

V.A@ $x=3$



2. $y = \frac{12x+24}{x^2+2x} = \frac{12(x+2)}{x(x+2)}$

$\{x \neq -2, 0\}$

Hole @ $x=-2$

V.A @ $x=0$

Horizontal Asymptotes: ← highest exponent

To find a horizontal asymptotes, we focus on the degree of the numerator and the denominator!

① N = D H.A @ coefficients

② N < D BOBO! H.A@ y=0

③ N > D No H.A

Examples:

1. $y = \frac{(x-3)(x+4)^2}{(x-5)^3(x+2)^2}$ Degree 3 / Degree 5 H.A@ y=0

2. $y = \frac{4x^6+3x^4-4x^2}{3x}$ Degree 6 / Degree 1 No H.A

3. $y = \frac{6x-5}{3x+9}$ Degree 1 / Degree 1 H.A@ $y = \frac{6}{3} = 2$

EVERYTHING ALL TOGETHER:

Find the domain, VA, holes, and HA for each Rational Function.

1. $y = \frac{5x^2+20x}{x^2+11x+28} = \frac{5x(x+4)}{(x+4)(x+7)}$

Restrictions $\{x \neq -4, -7\}$

① Domain: All real #'s except -4, -7

② V.A @ $x = -7$

③ Hole @ $x = -4$

④ H.A @ $y = 5$

Degree 2 / Degree 2 $\Rightarrow \frac{5}{1}$

$\mathbb{R}$ - means all real #'s

2. $y = \frac{2x^2+10x+12}{x^3+3x^2+2x} = \frac{2(x+3)(x+2)}{x(x+2)(x+1)}$

Restrictions: $\{x \neq -1, -2, 0\}$

① Domain: All real #s except -1, -2, 0

② V.A@ $x = -1, 0$

③ Hole @ $x = -2$

④ H.A@ $y = 0$

BOBO!