

Basic Probability
Desired Outcome
Total Outcome

Probability of an Event: $P(E) =$

Example 1: What is the probability that a dice lands on a 4?

$\{1, 2, 3, 4, 5, 6\}$ $P(4) = \frac{1}{6}$

Example 2: What is the probability that a dice lands on an odd number?

$P(\text{odd}) = \frac{3}{6} \rightarrow \frac{1}{2}$

Example 3: An experiment consists of tossing three coins.

1. List the sample space for the outcomes of the experiment.

$S = \{HHH, HHT, HTH, HTT, THT, TTH, THT, TTT\}$

2. Find the following probabilities:

a. $P(\text{all heads}) = \frac{1}{8}$

b. $P(\text{two tails}) = \frac{3}{8}$

c. $P(\text{no heads}) = \frac{1}{8}$

d. $P(\text{at least one tail}) = \frac{7}{8}$

The probabilities of all possible outcomes must add to 100% or 1

Thus $P(A) = \underline{1 - P(A^c)}$ and $P(A^c) = \underline{1 - P(A)}$

e. How could you use complements to find d?

$$\underline{1 - P(\text{no tails}) = 1 - \frac{1}{8} = \left(\frac{7}{8}\right)}$$

Example 4: A bag contains six red marbles, four blue marbles, two yellow marbles and 3 white marbles. One marble is drawn at random.

$\{R, B, Y, W\}$

1. What is the total number of marbles? $6+4+2+3=15$

2. Find the following probabilities:

a. P(red) $\frac{6}{15} \rightarrow \frac{2}{5}$

b. P(blue or white) $\frac{7}{15}$

c. P(not yellow) $\frac{13}{15}$

d. Show two ways to do part c.

① $1 - \frac{2}{5} = \frac{13}{15}$ ② $\frac{6+4+3}{15} = \frac{13}{15}$

Mutually Exclusive Events

Suppose you are rolling a six-sided die. What is the probability that you roll an A odd number or you roll a B 2?

- Can these both occur at the same time? Why or why not?

No, 2 is not an odd #

Mutually Exclusive Events:

Two or more events that can't occur at the same time

- The probability of two mutually exclusive events occurring at the **same time**, $P(A \text{ and } B)$, is Zero!

To find the probability of one of two mutually exclusive events occurring, use the following formula:

$$P(A \text{ or } B) = P(A) + P(B)$$

Examples:

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

A

1. If you randomly chose one of the integers 1 – 10, what is the probability of choosing either an odd number or an even number?

B

Are these mutually exclusive events? Why or why not? Yes, even \neq odd

Complete the following statement: $P(\text{odd or even}) = P(\text{odd}) + P(\text{even})$

Now fill in with numbers: $P(\text{odd or even}) = \frac{5}{10} + \frac{5}{10} = \frac{10}{10} \rightarrow 1$

Does this answer make sense? because all #'s 1-10 are either even or odd

2. Two fair dice are rolled. What is the probability of getting a sum less than 7 or a sum equal to 10?

Are these events mutually exclusive? Yes A B

Sometimes using a table of outcomes is useful. Complete the following table using the sums of two dice:

Roll a...	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(\text{getting a sum less than 7 OR sum of 10}) = P(<7) + P(10)$

$$\frac{15}{36} + \frac{3}{36} = \frac{18}{36} \rightarrow \left(\frac{1}{2}\right)$$

Mutually Inclusive Events

Suppose you are rolling a six-sided die. What is the probability that you roll an A B
odd number or a number less than 4?

- Can these both occur at the same time? If so, when?

Yes! # 1 and 3

Mutually Inclusive Events:

Two or more events that could
occur at the same time

Probability of the Union of Two Events: The Addition Rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

*** Must subtract off intersection or ***
We will double count

Examples:

1. What is the probability of choosing a card from a deck of cards that is a club or a ten?

$P(\text{choosing a club or a ten}) = \frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} \rightarrow \frac{4}{13}$

$P(A) = \frac{13}{52}$ $P(B) = \frac{4}{52}$ $P(A \cap B) = \frac{1}{52}$

\overline{A} \overline{B}

2. What is the probability of choosing a number from 1 to 10 that is less than 5 or odd?

$$P(A) = \frac{4}{10}$$

$$P(B) = \frac{5}{10}$$

$$P(A \cap B) = \frac{2}{10}$$

$$\frac{4}{10} + \frac{5}{10} - \frac{2}{10} = \frac{7}{10}$$