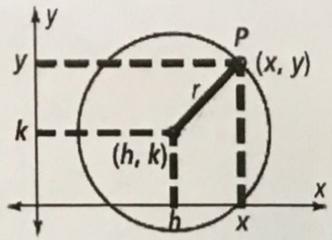
6.2 Equations of Circles

SWBAT graph circles on the coordinate plane and write the equations of circles in standard form.

Standard Form of Circles

$(x-h)^2+(y-k)^2=r^2$

Center:	Radius:	Point on the circle:
(h,k)	r	(x,y)



diameter$(\frac{1}{2})$ = radius

Example 1: Write the equation of a circle with the given information.

a) Center (0,0), Radius=10

h = 0 k = 0 r = 10

$(x-0)^2+(y-0)^2=10^2$

$\boxed{x^2+y^2=100}$

b) Center (2, 3), Diameter=12

h = 2 k = 3 r = 6

$(x-2)^2+(y-3)^2=6^2$

$\boxed{(x-2)^2+(y-3)^2=36}$

Example 2: Determine the center and radius of a circle the given equation.

a) $x^2+y^2=\frac{9}{4}$ $\sqrt{\frac{9}{4}}$

$\boxed{\text{center}: (0,0)\ r=\frac{3}{2}}$

b) $(x+3)^2+(y-5)^2=81$ $\sqrt{81}$

$\boxed{\text{center}: (-3,5)\ r=9}$

c) $(x+4)^2+(y+6)^2=1$ $\sqrt{1}$

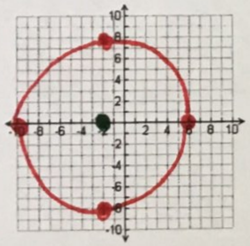
$\boxed{\text{center}: (-4,-6)\ r=1}$

Example 3: Use the center and the radius to graph each circle.

a) $(x+2)^2+y^2=64$

Center: $(-2,0)$

Radius: 8

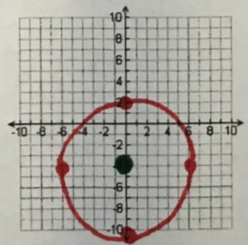


$\sqrt{64}=8$

b) $x^2+(y+4)^2=36$

Center: $(0,-4)$

Radius: 6



$\sqrt{36}=6$

☆ Plot the center first and then use the radius to get 4 other points.

Writing an Equation with a Pass-Thru Point

Step 1:	Substitute the center (h, k) into the equation
Step 2:	Substitute the "pass through point (x, y)" into the equation for x and y.
Step 3:	Simplify and solve for r^2.
Step 4:	Substitute r^2 back into the equation from Step 1.

$\boxed{(x-2)^2+(y-5)^2=45}$

Example 4: Write the equation of a circle with a given center (2, 5) that passes through the point (5, -1).

$(5-2)^2+(-1-5)^2=r^2$

$(3)^2+(-6)^2=r^2$

$9+36=r^2$

$45=r^2$

Writing an Equation with Two Points on the Circle	Midpoint Formula
Find the midpoint (radius) between the two endpoints, and then follow steps 1-4.	☆ Find the center ☆ $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Example 5: Write the equation of a circle with endpoints of diameter at (-6, 5) and (4, -3).

$\left(\frac{-6+4}{2}, \frac{5+-3}{2}\right)=(-1,1)$ center

$r^2=41$

$\boxed{(x+1)^2+(y-1)^2=41}$

Writing the Equation of a Circle in Standard Form

Step 1:	Group x's and group y's together.
Step 2:	Move any constants to the right side of the equation. ☆ No variables
Step 3:	Use complete the square to make a perfect square trinomial for the x's and then again for the y's. *Remember, whatever you do to one side of the equation, you must do to the other!
Step 4:	Simplify factors into standard form of a circle!

Example 5: Write the equation of a circle in standard form. Then, state the center and the radius.

a) $x^2+y^2+4x-8y+16=0$

b) $x^2+y^2+6x-4y=0$

c) $x^2+y^2-6x-2y+4=0$

d) $x^2+y^2+8x-10y-4=0$

a) $x^2+y^2+4x-8y+16=0$

Step 1: $\underbrace{x^2+4x}+\underbrace{y^2-8y}+16=0$

$-16 \quad -16$

Step 2: $x^2+4x+y^2-8y=-16$

Step 3: $x^2+4x+\underline{4}+y^2-8y+\underline{16}=-16+4+16$

$\frac{4}{2}=(2)^2$ $\quad$ $\frac{-8}{2}=(-4)^2$ $\quad$ $=4$

* Be sure to add it to both sides *

Step 4: $\boxed{(x+2)^2+(y-4)^2=4}$

center: $(-2, 4)$ radius $=2$

b) $x^2+y^2+6x-4y=0$

Step 1: $x^2+6x+y^2-4y=0$

Step 2: No constants!

Step 3: $x^2+6x+\underline{9}+y^2-4y+\underline{4}=0$

$\frac{6}{2}=(3)^2$ $\quad$ $\frac{-4}{2}=(-2)^2$

$+9$
$+4$
13

Step 4: $\boxed{(x+3)^2+(y-2)^2=13}$

center: $(-3, 2)$ $r=\sqrt{13}$

c) $x^2 - 6x + \underline{\quad 9 \quad} + y^2 - 2y + \underline{\quad 1 \quad} = -4 + 9 + 1$

$\frac{-6}{2} = (-3)^2$ $\quad\quad$ $\frac{-2}{2} = (-1)^2$

center: $(3,1)$ $\quad r = \sqrt{6}$

$(x-3)^2 + (y-1)^2 = 6$

d) $x^2 + 8x + \underline{\quad 16 \quad} + y^2 - 10y + \underline{\quad 25 \quad} = 4 + 16 + 25$

$\frac{8}{2} = (4)^2$ $\quad\quad$ $\frac{-10}{2} = (-5)^2$

$(x+4)^2 + (y-5)^2 = 45$ $\quad$ center: $(-4, 5)$ $\quad r = \sqrt{45}$ or $3\sqrt{5}$