Length of a Circular Arc

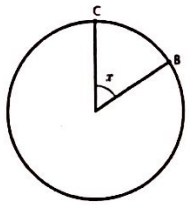
Arcs have two properties. They have a measurable curvature based upon the corresponding central angle (measure of arc = measure of central angle). Arcs also have a length as a portion of the circumference.

$$\frac{\text{portion of circle}}{\text{whole circle}} = \frac{\text{central angle in degrees}}{360°} = \frac{\text{central angle in radians}}{2\pi} = \frac{\text{arc length}}{\text{circumference}}$$

$$\frac{x°}{360°} = \frac{\text{length } \overset{\frown}{CB}}{2\pi r}$$

-or -

$$\frac{x \text{ (radians)}}{2\pi} = \frac{\text{length } \overset{\frown}{CB}}{2\pi r}$$



Remember:
- circumference of a circle = $2\pi r$

For a central angle **θ** in radians, and arc length **s** - the proportion can be simplified to a formula:

$$\frac{\theta}{2\pi} = \frac{s}{2\pi r}$$
$$s2\pi = \theta 2\pi r$$
$$s = \theta r$$

Length of an Arc: $s = r\theta$ for θ in radians

$S = r\theta\left(\frac{\pi}{180}\right)$

* use if in degrees

* only if θ is in radians

Examples:

1) For a central angle of $\pi/6$ in a circle of radius 10 cm, find the length of the intercepted arc.

2) For a central angle of $4\pi/7$ in a circle of radius 8 in, find the length of the intercepted arc.

3) For a central angle of 40° in a circle of radius 6 cm, find the length of the intercepted arc.

4.) Find the degree measure to the nearest tenth of the central angle in a circle that has an arc length of 87 and a radius of 16 cm.

Area of a Sector

Sector of a circle: a region bounded by a central angle and the intercepted arc

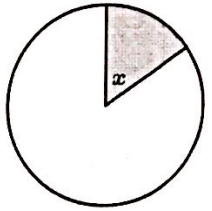
Sectors have an area as a portion of the total area of the circle.

$$\frac{\text{portion of circle}}{\text{whole circle}} = \frac{\text{central angle in degrees}}{360°} = \frac{\text{central angle in radians}}{2\pi} = \frac{\text{area of sector}}{\text{area of circle}}$$

$$\frac{x°}{360°} = \frac{\text{area of sector}}{\pi r^2}$$

-or -

$$\frac{x \text{ (radians)}}{2\pi} = \frac{\text{area of sector}}{\pi r^2}$$



Remember:
- area of a circle = πr^2

For a central angle **θ** in radians, and area of sector **A**, the proportion can be simplified to a formula:

$$\frac{\theta}{2\pi} = \frac{A}{\pi r^2}$$
$$A2\pi = \theta\pi r^2$$
$$A = \frac{1}{2}\theta r^2$$

Area of a Circular Sector: $A = \frac{1}{2}r^2\theta$ for θ in radians

$A = \frac{1}{2}r^2\theta\left(\frac{\pi}{180}\right)$

* use if in degrees

* θ is in radians

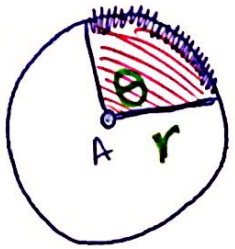
Examples:

5) Find the area of the sector of the circle that has a central angle measure of $\pi/6$ and a radius of 14 cm.

6) Find the area of the sector of the circle that has a central angle measure of 60° and a radius of 9 in.

HONORS

7) A sector has arc length 12 cm and a central angle measuring 1.25 radians Find the radius of the circle and the area of the sector.



Arc length = crust

Area of Sector = cheese

θ = central angle

r = radius

Ex 1) $S = r\theta \quad \theta = \frac{\pi}{6} \quad r = 10cm$

$S = 10\left(\frac{\pi}{6}\right) = \frac{10\pi}{6} = \boxed{\frac{5\pi}{3}\ cm}$

2) $S = r\theta \quad r = 8in \quad \theta = \frac{4\pi}{7}$

$S = 8\left(\frac{4\pi}{7}\right) = \boxed{\frac{32\pi}{7}\ in}$

3) $S = r\theta\left(\frac{\pi}{180}\right) \quad r = 6cm \quad \theta = 40°$

$S = 6(40)\left(\frac{\pi}{180}\right) = \frac{240\pi}{180}\ cm$

$= \boxed{\frac{4\pi}{3}\ cm}$

4) $S = r\theta\left(\frac{\pi}{180}\right) \quad r = 16cm \quad S = 87\ cm$

$180(87) = \left[16(\theta)\left(\frac{\pi}{180}\right)\right]180$

$\frac{15660}{16\pi} = \frac{16\pi(\theta)}{16\pi}$

$\boxed{\theta = 311.5°}$

Ex 5) $A = \frac{1}{2}r^2\theta \quad \theta = \frac{\pi}{6} \quad r = 14cm$

$A = \frac{1}{2}(14)^2\left(\frac{\pi}{6}\right) = \frac{196\pi}{12}$

$= \boxed{\frac{49\pi}{3}\ cm^2}$

6) $A = \frac{1}{2}r^2\theta\left(\frac{\pi}{180}\right) \quad \theta = 60° \quad r = 9in$

$A = \frac{1}{2}(9)^2(60)\left(\frac{\pi}{180}\right) = \frac{4860\pi}{360}$

$= \boxed{\frac{27\pi}{2}\ in^2}$

7) $S = r\theta$

$12 = r(1.25)$

$\boxed{r = 9.6cm}$

$A = \frac{1}{2}r^2\theta$

$A = \frac{1}{2}(9.6)^2(1.25)$

$\boxed{A = 57.6cm^2}$