Name _____ Math 3 –Solving Polynomials

List all the ways we have discussed to find the zeroes of a polynomials below:

- look at the graph on the x-axis
- look at the equation in factored form
- long division and synthetic division

A polynomial equation has the same number of solutions (zeroes) as its exponent (degree).

How many solutions does each poly have?

1) $4x^3 - 4x + 4 = 0$ — 3 zeros

2) $3x^2 + x + 3 = 0$ — 2 zeros

3) $5x^4 + 7x - 1 = 0$ — 4 zeros

4) Graph $x^3 - x^2 - 4x + 24 = 0$ How many zeroes should it have? 3 How many do you see? 1

Solve by graphing only works for **real** solutions.

In examples like these follow these steps:

1) FIND the one real solution, turn it into a factor. $x = -3$ $(x+3)$

2) Use division to turn the cubic into a quadratic. ⇒ I can use synthetic division since its in the form $x+3$.

3) Use one of the other methods to find

Now, finish solving $x^3 - x^2 - 4x + 24 = 0$

$$\begin{array}{r|rrrr} -3 & 1 & -1 & -4 & 24 \\ & \downarrow & -3 & 12 & -24 \\ \hline & 1 & -4 & 8 & 0 \end{array}$$

$x^2 - 4x + 8$

$a = 1 \quad b = -4 \quad c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-16}}{2} = \frac{4 \pm i\sqrt{16}}{2}$$

$$x = \frac{\overset{\div 2}{4} \pm \overset{\div 2}{4i}}{2 \div 2} = 2 \pm 2i$$

Zeros: $x = -3$, $x = 2+2i$, $x = 2-2i$

5) Solve $x^3 - x^2 - 2x - 12 = 0$ I should see 3 zeros.

$x = 3 \Rightarrow (x-3)$

3	1	-1	-2	-12
	↓	3	6	12
	1	2	4	0

$x^2 + 2x + 4$

$a=1 \quad b=2 \quad c=4$

$$x = \frac{-(2) \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{-12}}{2} = \frac{-2 \pm i\sqrt{12}}{2}$$

$$= \frac{-2 \pm i2\sqrt{3}}{2} = \boxed{-1 \pm i\sqrt{3}}$$

12 = 3 × 4, 4 = 2 × 2

6) $2x^3 + 9x^2 + 25 = 0$ 3 zeros

$x = -5 \quad (x+5)$

-5	2	9	0	25
	↓	-10	5	-25
	2	-1	5	0

$2x^2 - x + 5$

$a=2 \quad b=-1 \quad c=5$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(5)}}{2(2)} = \frac{1 \pm \sqrt{-39}}{4}$$

$$= \boxed{\frac{1 \pm i\sqrt{39}}{4}}$$

zeros: $x = 3$, $x = -1 + i\sqrt{3}$, $x = -1 - i\sqrt{3}$

zeros: $x = -5$, $x = \frac{1 + i\sqrt{39}}{4}$, $x = \frac{1 - i\sqrt{39}}{4}$

7) $x^3 + 27 = 0$

8) $x^4 + 7x^2 + 6 = 0$

9) $x^4 - 3x^2 - 10 = 0$