

Polynomial: An expression in the form

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

when n is a positive integer.

Example

① $\frac{1}{2}x^2 + 2x + 1$

② $5x^5 + 3x - 5$

③ $6x^3 + 0.75x^2$

Non-Examples

① x^{-1}

② $5x^{1/3} - 8$

③ 11^x

Terms: #'s and/or variables that are separated by + or - sign

Monomial
1 term

Binomial
2 terms

Trinomial
3 terms

Polynomial
(4+) terms

Degree: The highest exponent of the polynomial.

Polynomial Vocabulary

Zeros * Multiplicity

Extrema * End Behavior

Solving Polynomials

Rates of Change

Review

Long Division of Polynomials

$$\begin{array}{r} 20 \\ 5 \overline{) 100} \\ \underline{-10} \downarrow \\ 00 \\ \underline{-0} \\ 0 \end{array}$$

$$(20)(5) = 100$$

No Remainder!

$$\begin{array}{r} 107 \\ 25 \overline{) 2675} \\ \underline{-25} \downarrow \downarrow \\ 175 \\ \underline{-175} \\ 0 \end{array}$$

$$(107)(25) = 2675$$

Example 1

$$\frac{27x^3 + 9x^2 - 3x - 10}{3x - 2}$$

$$\begin{array}{r} 9x^2 + 9x + 5 \\ 3x - 2 \overline{) 27x^3 + 9x^2 - 3x - 10} \\ \underline{-27x^3 + 18x^2} \downarrow \\ 27x^2 - 3x \\ \underline{-27x^2 + 18x} \downarrow \\ 15x - 10 \\ \underline{-15x + 10} \\ 0 \end{array}$$

*When subtracting change the signs!

0 Remainder

$$\begin{aligned} 27x^3 + 9x^2 - 3x - 10 \\ = (3x - 2)(9x^2 + 9x + 5) \end{aligned}$$

Example 2

$$\frac{x^4 + 3x^3 - x^2 - x + 6}{x + 3}$$

$$\begin{array}{r} x^3 + 0x^2 - x + 2 \\ x+3 \overline{) x^4 + 3x^3 - x^2 - x + 6} \\ \underline{-x^4 + 3x^3} \\ 0 - x^2 - x \\ \underline{+x^2 + 3x} \\ 2x + 6 \\ \underline{-2x + 6} \\ 0 \end{array}$$

$$\begin{aligned} x^4 + 3x^3 - x^2 - x + 6 \\ = (x+3)(x^3 - x + 2) \end{aligned}$$

0 Remainder

Example 3

$$\frac{x^3 - 1}{x - 1}$$

$$\begin{array}{r} x^2 + x + 1 \\ x-1 \overline{) x^3 + 0x^2 + 0x - 1} \\ \underline{-x^3 + x^2} \\ x^2 + 0x \\ \underline{-x^2 + x} \\ x - 1 \\ \underline{-x + 1} \\ 0 \end{array}$$

$$\begin{aligned} x^3 - 1 \\ = (x-1)(x^2 + x + 1) \end{aligned}$$

0 Remainder

Example 4

$$\frac{x^3 - 4x^2 + 11}{x - 3}$$

$$\begin{array}{r} x^2 - x - 3 \\ x-3 \overline{) x^3 - 4x^2 + 0x + 11} \\ \underline{-x^3 + 3x^2} \\ -x^2 + 0x \\ \underline{+x^2 - 3x} \\ -3x + 11 \\ \underline{+3x - 9} \\ 2 \end{array}$$

$\boxed{2}$ Remainder

$$x^3 - 4x^2 + 11 = \left(x^2 - x - 3 + \frac{2}{x-3} \right) (x-3)$$

Remainder is numerator!