Example 2 Solve a rational equation

To solve a rational equation, multiply each term by the LCD to eliminate the fractions. Then solve for x. Remember to check for extraneous solutions.

A] $\frac{2}{3x}+\frac{1}{6}=\frac{4}{3x}$

$\boxed{LCD: 6x}$

$\overset{2}{\cancel{6x}}\left(\frac{2}{\cancel{3x}}\right)+\overset{x}{\cancel{6}x}\left(\frac{1}{\cancel{6}}\right)=\left(\frac{4}{\cancel{3x}}\right)\overset{2}{\cancel{6x}}$

$4+x=8$

$-4 \qquad -4$

$\boxed{x=4}$

Example 2 Solve a rational equation

To solve a rational equation, multiply each term by the LCD to eliminate the fractions. Then solve for x. Remember to check for extraneous solutions.

B] $\frac{1}{x-2}+2=\frac{3x}{x+2}$

$$\frac{1}{(x-2)}+\frac{2}{1}=\frac{3x}{(x+2)}$$

$\boxed{\underline{\text{LCD:}}\ (x-2)(x+2)}$

$$x+2+2x^2+8=3x^2-6x$$

$$\cancel{x}+\cancel{10}+\cancel{2x^2}=3x^2-6x$$

$$-\cancel{x}\ -\cancel{10}\ -\cancel{2x^2}\quad -2x^2-x\ -10$$

$$0=x^2-7x-10$$

Example 3 Rational equations with extraneous solutions

Extraneous solutions are answers that are algebraically correct but do not check in the original problem. Remember that you can never divide by zero, so any value of x that makes a denominator in the original problem equal zero is restricted from the domain.

A] $\frac{2}{x-3}=\frac{1}{x^2-2x-3}$

Cross
Multiply

Example 3 Rational equations with extraneous solutions

Extraneous solutions are answers that are algebraically correct but do not check in the original problem. Remember that you can never divide by zero, so any value of x that makes a denominator in the original problem equal zero is restricted from the domain.

B] $\frac{2}{x-3}+\frac{1}{x}=\frac{x-1}{x-3}$

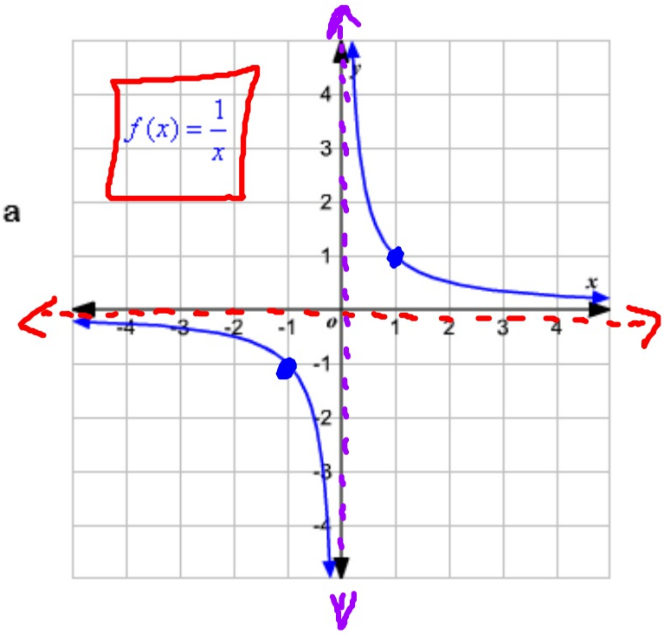
$$\frac{2}{(x-3)}+\frac{1}{(x)}=\frac{x-1}{x-3}$$

LCD:

$$\boxed{(x-3)x}$$

The Graph to the right is the Parent Function for Rational Functions also known as the Inverse Function.

Just like a Quadratic Function graphs as a parabola, the Rational Function has a special name as well. Its shape is called a ________________



Characteristics of $f(x) = \frac{1}{x}$

Horiztonal Asymptote at $y=0$

Vertical Asymptotes at $x=0$

Domain: $x \neq 0$ Range: $y \neq 0$

Passes through (-1 , -1) and (1 , 1)

Define "asymptote" A line that the graph approaches but never crosses or touches

Key Characteristics of the Rational Function

$$\boxed{y = \frac{a}{x-h} + k}$$

a≥1 the function Stretch
from left to right

a<0 Reflect the graph in the x-axis
so the function decrease from left to right

a/1 is the slope from the intersection of the asymptotes to the points one unit left and right

+k shifts the graph up
-k shifts the graph down

Horizontal asymptote y=k.
The range is y≠k.

(x-h) shifts the graph right
(x+h) shifts the graph left

Vertical asymptote x=h.
The domain is x≠h.

If a is fraction,
it Compresses

Rational functions are still transformed around the coordinate plane, and follow the same function transformation rules that we have been following in this course.

1. $y = \frac{1}{x-3} + 4$

$Y = \frac{1}{x-3} + 4$

Transformations: Right 3, Up 4

VA: $X = 3$

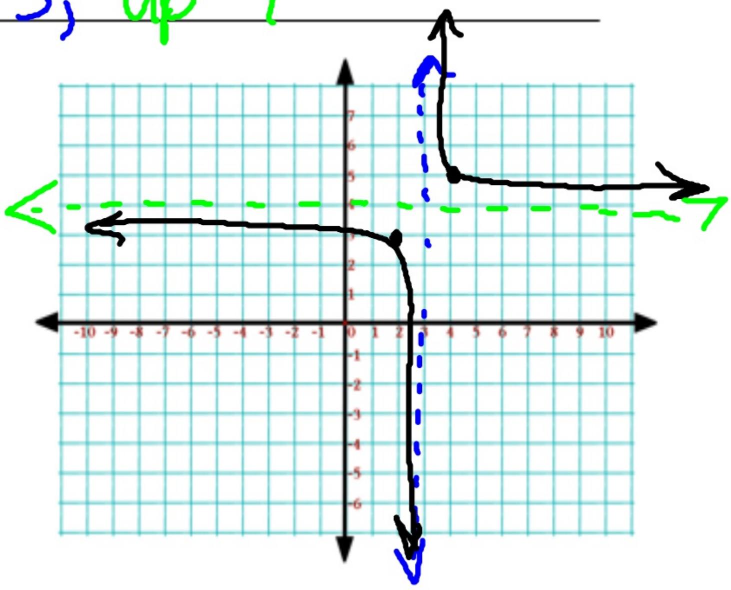
HA: $Y = 4$

Quadrants: I, III

Distance: ____________

Domain: $X \neq 3$

Range: $Y \neq 4$



2. $y = -\frac{4}{x+1}$

$Y = \frac{-4}{x+1}$

Transformations: Reflect, Stretch by 4, left 1

VA: $x = -1$

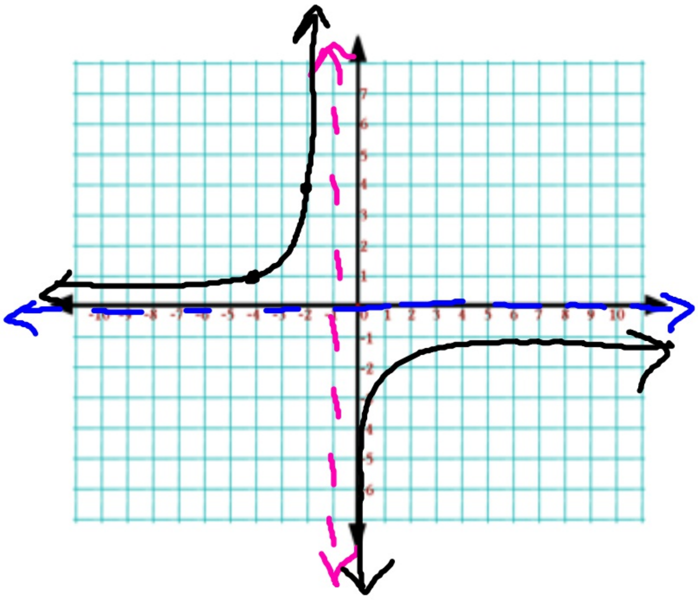
HA: $y = 0$

Quadrants: II, IV

Distance:

Domain: $x \neq -1$

Range: $y \neq 0$



3. Write the equation of $y = -\frac{1}{x}$ that has asymptotes at x= -4, and y= 5.

Answer: $y = \frac{-1}{x+4} + 5$

V.A

(denominator)

H.A

(added to the outside)