

Start Here: $4(x - 3)^2 - 36 = 0$

$$4(x - 3)^2 = 36$$

$$(x - 3)^2 = 9$$

$$\sqrt{(x - 3)^2} = \pm\sqrt{9}$$

$$x - 3 = 3$$

$$x = 6$$

$$x - 3 = -3$$

$$x = 0$$

Unit 2B: Solving Quadratic Equations

Quadratic Equations: A lot of times we say "Solve" in math, when we really mean something else, like simplify, factor, multiply, etc. You can only solve equations, which involves finding the values that make the equation True.

To solve a quadratic equation, you must have it set equal to zero.

$$ax^2 + bx + c = 0$$

When you solve a polynomial, the values you find are called:

X-intercept, zeroes, solutions
roots

EX7: Solve $3x^2 - 2x - 8 = 0$

$$3x^2 - 2x - 8 = 0$$

$$(3x+2)(x-2) = 0$$

$$3 \cdot -8 = -24$$

$$\begin{array}{c} \swarrow \quad \searrow \\ \underline{-6x} + \underline{4x} \end{array}$$

	<u>3x</u>	<u>2</u>
<u>x</u>	$3x^2$	$2x$
<u>-2</u>	$-6x$	-8

24 · 1

2 · 12

3 · 8

6 · 4

Solving by Factoring:

Recall that the **Factored Form** of a quadratic represents the **x-intercepts** of the graph of its parabola. We have said that the numbers in the factors appears as their "opposites" in the x-intercepts. Today we will delve into this idea more rigorously.

Example Without graphing, what are the x-intercepts of the quadratic function $y = (x - 8)(x + 3)$?

$(8, 0)$ $(-3, 0)$

We can call these numbers the *x-intercepts*, but there are other names for them as well.

x-intercept = Roots = Zeros = Solutions

When we say "solve" a quadratic equation, we mean "find the solutions" or "find the x-intercepts".

Example Solve the quadratic equation $y = (x + 4)(x - 1)$.

$$0 = (x + 4)(x - 1)$$

$$(-4, 0) \text{ and } (1, 0)$$

Solve
means
look for
x-intercept!

Example Solve the quadratic equation $y = (x - 6)^2$.

$$0 = (x - 6)(x - 6)$$

$$(6, 0) \text{ and } (6, 0)$$

Standard

Example Solve the quadratic equation $y = x^2 + 14x + 40$ by factoring.

$$0 = 1x^2 + 14x + 40$$

$$\begin{array}{r} 40 \\ \wedge \\ 10x + 4x = 14x \end{array}$$

$$0 = (x+10)(x+4)$$

$$\begin{array}{|l} (-10, 0) \\ (-4, 0) \end{array}$$

	<u>x</u>	<u>10</u>
<u>x</u>	x^2	$10x$
<u>4</u>	$4x$	40

Example Solve the quadratic equation $y = x^2 - 4x - 5$ by factoring.

$$(-1, 0)$$

$$(5, 0)$$

$$0 = x^2 - 4x - 5$$

$$0 = (x+1)(x-5)$$

Zero Product Property:

The zero product property is a tool that we use to help us solve equations algebraically. It can be applied to integers, algebraic terms, and polynomials. Let's take a look at some examples.

Integers

- $5 * 0 = 0$

- $0 * 5 = 0$

Algebraic terms

- If $x * 5 = 0$ then $x = 0$

- If $x * y = 0$ then $x = 0$ or $y = 0$

Polynomials

- If $(x + 2) * (x + 4) = 0$

then $(x + 2) = 0$ or $(x + 4) = 0$
 $x = -2$ or $x = -4$

- If $(3x - 1) * (5x + 9) = 0$

$$\begin{array}{r} 3x - 1 = 0 \\ \hline +1 \quad +1 \\ \hline 3x = 1 \\ \frac{3x}{3} = \frac{1}{3} \end{array}$$

then $(3x - 1) = 0$ or $(5x + 9) = 0$
 $x = \frac{1}{3}$ or $x = -\frac{9}{5}$

Summary: If $a * b = 0$ then either $a = 0$ or $b = 0$, or both, $a \text{ and } b = 0$

Think: Why are the x-intercepts, or zeros, of a quadratic equation the "opposites" of the numbers in the factors?

They are "opp" b/c when we solve for x,
we have to undo PEMDAS

Example: Solve the quadratic equation $y = 8x^2 - 2x - 3$ by factoring.

$$0 = 8x^2 - 2x - 3$$

$$\begin{array}{r} -24 \\ \swarrow \quad \searrow \\ -6 \quad +4 = -2 \end{array}$$

$$\left(\frac{3}{4}, 0 \right) \left(-\frac{1}{2}, 0 \right)$$

$$\begin{array}{r} 4x - 3 = 0 \\ +3 \quad +3 \\ \hline 4x = 3 \\ \frac{4x}{4} = \frac{3}{4} \end{array}$$

$$x = \frac{3}{4}$$

$$\begin{array}{r} 2x + 1 = 0 \\ \quad -1 \quad -1 \\ \hline 2x = -1 \\ \frac{2x}{2} = \frac{-1}{2} \end{array}$$

$$x = -\frac{1}{2}$$

Example: Solve the quadratic equation $y = 10x^2 + 11x - 35$ by factoring.

$$\begin{array}{r}
 -350 \\
 \diagup \quad \diagdown \\
 25 \quad + \quad -14 = 11
 \end{array}$$

- 350 · 1
- 2 · 175
- 5 · 70
- 35 · 10
- 25 · 14

	$2x$	5
$5x$	$10x^2$	$25x$
-7	$-14x$	-35

$$\begin{array}{l}
 (5x - 7) = 0 \\
 \frac{+7}{5} \quad \frac{+7}{5} \\
 \frac{5x}{5} = \frac{7}{5} \\
 x = 7/5
 \end{array}$$

$$\begin{array}{l}
 2x + 5 = 0 \\
 \frac{-5}{2} \quad \frac{-5}{2} \\
 \frac{2x}{2} = \frac{-5}{2} \\
 x = -5/2
 \end{array}$$

$\left(\frac{7}{5}, 0\right) \quad \left(-\frac{5}{2}, 0\right)$

You Try! Practice

Use the zero product property to solve the following quadratic equations.

1. $y = (x - 2)(x + 3)$

2. $y = x^2 + 5x - 6$

3. $y = x^2 + 4x - 32$

4. $y = (2x - 1)(3x + 4)$

Homework is Page 2 in Packet