Solving Exponential Equations

For the last two days you have seen properties of logarithms, how to convert between logarithmic and exponential form and how to solve logarithmic equations. Today, we will solve exponential equations. You actually already have the skills to do this!

Example 1: $2^x = 8$

This is asking, "2 raised to what power would equal 16?" You could probably do this example in your head. $x = 3$ since $2^3 = 8$.

However, not every example will be so easy to do. How can we solve Example 1 using logarithms?

Well, we just have to remember those two things:

#1 base of log = base of exponent

#2 log = exponent

So then: $2^x = 8$ (exponent, base)

$\log_2 8 = x$

we can put this in the calculator (Alpha) (window) (#5) $x = 3$

Example 2: $7^x = 343$

$\log_7 243 = x$

$x = 2.82$

Example 3: $10^x = 500$

$\log_{10} 500 = x$

OR

$\log 500 = x$

$x = 2.70$

Example 4: $e^x = 5$

* Remember ln has a base of e. e is just a # similar to π

$\ln_e 5 = x$ OR $\ln 5 = x$

$x = 1.61$

Let's make things a little more interesting now!

Example 5: $3^{x-2} = 81$ (exponent, base)

$\log_3 81 = x - 2$

Put this in your calculator first since its just a #

$4 = x - 2$

$+2 \quad +2$

$x = 6$

Example 6: $2^{3x} + 6 = 10$

* Before we change forms we have to get the base by itself.

$2^{3x} + 6 = 10$

$-6 \quad -6$

$2^{3x} = 4$ ← Now I can solve regularly

$\log_2 4 = 3x$

$2 = 3x$

$x = \frac{2}{3}$

Example 7: $5 = 7 - 2 \cdot 4^{6x-38}$

* I have to get 4^{6x-38} by itself first!

$5 = 7 - 2 \cdot 4^{6x-38}$

$-7 \quad -7$

$-2 = -2 \cdot 4^{6x-38}$

↑ multiplication so ÷ by −2

$1 = 4^{6x-38}$

$\log_4 1 = 6x - 38$

$0 = 6x - 38$

$\frac{38}{6} = \frac{6x}{6}$

$x = 6.\overline{3}$