Unit 4 Review - Polynomials NAME Key

Polynomial Division

Divide using either long division or synthetic division (when possible).

1. $(9x^3 - 2x^2 + 5x + 4) \div (x - 3)$

$9x^2 + 25x + 80 + \frac{244}{x-3}$

2. $(6x^3 + 19x^2 + 7x - 12) \div (2x + 3)$.

$3x^2 + 5x - 4$

3. $(12x^3 - 7x^2 - 38x + 35) \div (4x - 5)$

$3x^2 + 2x + 7 + \frac{70}{4x-5}$

4. $(x^4 + 7x^3 - 6x + 2) \div (x + 4)$

$x^3 + 3x^2 - 12x + 42 - \frac{166}{x+4}$

Remainder/Factor Theorem

Determine which are factors of $2x^{91} - x^{90} - 10x^{89}$.

5. $3x + 1$

No

6. $2x - 5$

Yes

7. $x + 2$

Yes

Polynomial Vocabulary

Classify each polynomial by the degree and by the number of terms.

8. $7x^3 - 2x$

Cubic Binomial

9. $-10x^4 - 3x^3 + 2$

Quartic Trinomial

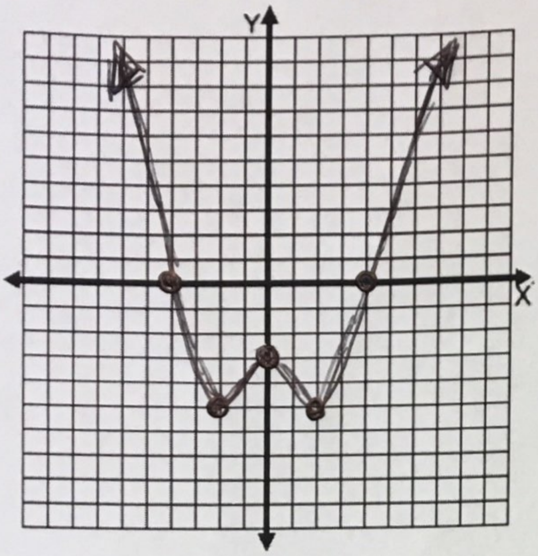
10. 7

Constant Monomial

Zeroes and Multiplicity, Extrema, Intervals for Increasing/Decreasing/Positive/Negative

For each graph and equation, determine all key features.

11.



Zeroes: X = -4 m1, x = 4 mult 1

Extrema: Abs Min @ (-2,-5) (2,-5) Rel Min (0,-3)

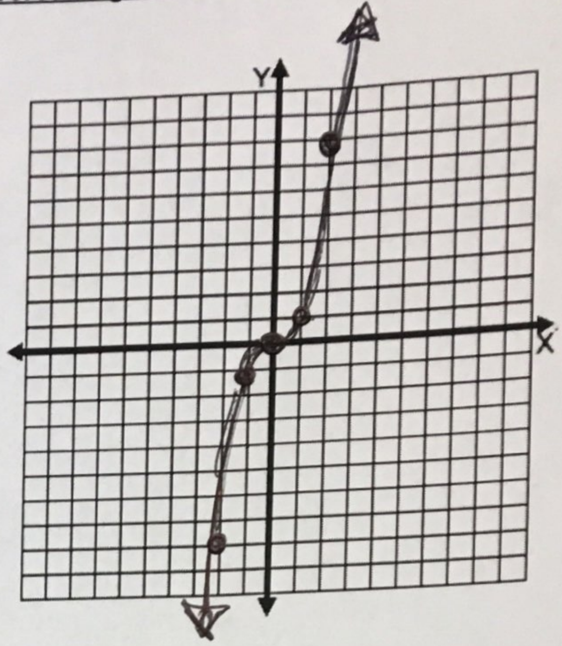
Pos/Neg: P: $(-\infty,-4)$ $(4,\infty)$ N: (-4,4)

Inc/Dec: I: (-2,0) $(2,\infty)$ D: $(-\infty,-2)$ (0,2)

End Behavior: $x \to -\infty, y \to \infty$ and $x \to \infty, y \to \infty$

Degree: 4th

12.



Zeroes: X = 0 mult. 3

Extrema: None

Pos/Neg: P: $(0,\infty)$ N: $(-\infty,0)$

Inc/Dec: I: $(-\infty,\infty)$

End Behavior: $x \to -\infty, y \to -\infty$ and $x \to \infty, y \to \infty$

Degree: 3rd

13. $y = -2(x+1)^2(3x-1)$

Zeroes: X = -1 m2, $x = \frac{1}{3}$ m1

Extrema: Rel Min (-1,0) Rel Max (0,2)

Pos/Neg: P: $(-\infty, \frac{1}{3})$ N: $(\frac{1}{3}, \infty)$

Inc/Dec: I: (-1,0) D: $(-\infty,-1)$ and $(0,\infty)$

End Behavior: $x \to -\infty, y \to \infty$ and $x \to \infty, y \to -\infty$

Degree: 3rd

14. $y = x^3(x-2)(x-3)$

Zeroes: X = 0 m3, x = 2 m1, x = 3 m1

Extrema: Rel Max (1.37, 2.64) Rel Max (2.63, -4.24)

Pos/Neg: P: (0,2)$(3,\infty)$ N: $(-\infty,0)$ (2,3)

Inc/Dec: I: $(-\infty, 1.37)$ $(2.63,\infty)$ D: (1.37, 2.63)

End Behavior: $x \to -\infty, y \to -\infty$ and $x \to \infty$ $y \to \infty$

Degree: 5th

Solve Polynomials

Determine all real and complex solutions.

15. $x^3 - 5x^2 + 3x - 15 = 0$

$X = 5, \sqrt{3}i, -\sqrt{3}i$

16. $x^4 - 3x^3 - 24x^2 + 80x = 0$

$X = 0, 4, -5$

17. $x^3 + 64 = 0$

$X = -4, 2+2i\sqrt{3}, 2-2i\sqrt{3}$

18. $x^3 + 5x^2 + 10x + 24 = 0$

$X = -4, \frac{-1+i\sqrt{23}}{2}, \frac{-1-i\sqrt{23}}{2}$

Applications

19. The weight of an ideal round-cut diamond can be modeled by $w = 0.0074d^3 - 0.087d^2 + 0.32d$, where w is the diamond's weight (in carats) and d is its diameter (in millimeters). According to the model, what is the weight of a diamond with a diameter of 12 millimeters?

384.26 carats

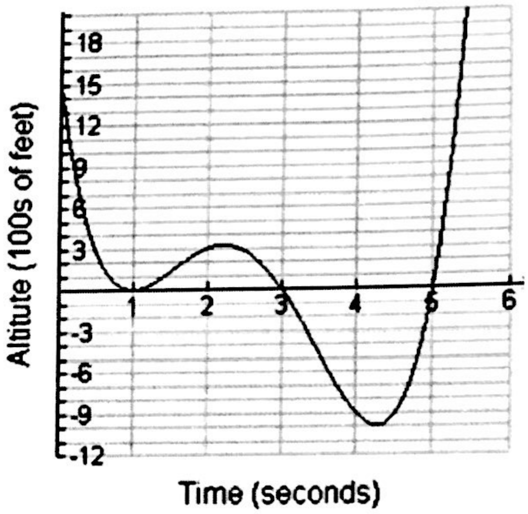
20. The profit P (in millions of dollars) for a t-shirt manufacturer can be modeled by $P = -x^3 + 5x^2 + 9x$, where x is the number of t-shirts produced (in millions). Currently, the company produces 5 million t-shirts and makes a profit of $45,000,000. What lesser number of t-shirts could the company produce and still make the same profit?

3 million t-shirts

21. A box has a height of $x-4$ inches and a length of $x+3$ inches. If the volume of the box is $2x^3-3x^2-23x+12$ cubic inches, determine the width of the box.

2x-1 inches

22. When fighter pilots train for dog-fighting, a "hard-deck" is usually established below which no competitive activity can take place. The polynomial graph given shows Maverick's altitude (y in 100s of feet) above and below this hard-deck during a 5 second (x) interval.



a. What is the lowest possible degree of this polynomial?

4th

b. How many total seconds was Maverick above the hard-deck during the first 5 seconds?

3 seconds

c. After how many seconds is Maverick 300 feet above the hard-deck?

after 2 seconds

d. Determine the equation of the function in factored form.

$(x-1)^2(x-3)(x-5)$

Rates of Change

23. Find the average rate of change from ***x = −1*** to ***x = 3*** for each of the functions below.

a. $a(x)=2x+3$

2

b. $b(x)=x^2-2$

2

c. $c(x)=2^x-1$

1.88

d. Which function has the greatest average rate of change over the interval **[− 1, 3]**?

a(x) and b(x)

24. In general as $x\to\infty$, which function eventually grows at the fastest rate?

a. $a(x)=3x$

b. $b(x)=x^3$

c. $c(x)=3^x$